

Performance Analysis of Frequency Hopping Techniques in DBPSK Modulation System

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Abstract—With more and more developments in wireless communication, spread spectrum modulation techniques have been widely used in various modulation systems. Frequency hopping technique is one of the types of spread spectrum system. In this paper, frequency hopping techniques are compared in a DBPSK modulation system which employs non coherent reception. Fast frequency hopping and multicarrier frequency hopping methods are compared based on the probability of error equations. It has been found that multicarrier frequency hopping techniques completely outperforms the fast frequency hopping techniques.

Keywords—Hopping Techniques, Fast Frequency Hopping (FFH), Multicarrier Frequency Hopping (MCFH), Modulation, Differential Binary Phase Shift Keying (DBPSK), Probability of Error; Non Coherent Reception.

1 INTRODUCTION

Spread spectrum modulation techniques have become very popular in recent years. Spread spectrum enables a signal to be transmitted across a frequency band that is much wider than the bandwidth required by the information signal. Then the transmitter spreads the energy, originally concentrated in narrowband, across a number of frequency band channels on a wider electromagnetic spectrum [1]. In frequency hopping spread spectrum (FHSS) systems, transmitter hops between the available frequencies according to a specific algorithm, which can be random or preplanned. The transmitter works in synchronization with the receiver, which remains tuned to same center frequency as the transmitter. A short burst of data is transmitted on a narrowband. The transmitter then tunes to another frequency and transmits again. Hence the receiver is capable of hopping its frequency over a given bandwidth several times, transmitting on one frequency for a certain period of time, then hopping to another frequency and transmitting again [2]. FHSS systems have been widely used in military applications and to combat multipath fading [3]. Transmission diversity provides protection against multiple access interference and fading. For FHSS systems, the diversity can be realized in the form of fast frequency hopping (FFH) and multicarrier frequency hopping (MCFH) transmission. In a FFH system, the diversity is obtained by changing the transmit frequency more than once over one symbol duration. The transmit frequency can be selected from the entire dedicated frequency band. In a MCFH system, the total frequency band is partitioned into several disjoint sub

bands on which replicas of the same data are simultaneously transmitted. In contrast to the FFH system, in MCFH system, the frequency hopping rate is at most equal to symbol rate. The performance of the FFH systems has been widely studied over the past few decades. The MCFH system with binary phase shift keying (BPSK) modulation and coherent detection was first introduced in [4], while coherent detection provides somewhat superior performance compared to non-coherent case [5], coherent reception in some applications is not practical. Differential binary phase shift keying (DBPSK) modulation with non coherent demodulation is typically employed in FHSS systems [6, 7]. Hence in this paper, DBPSK modulation and non coherent demodulation are assumed to be employed for both FFH and MCFH systems

This paper is arranged into six sections. Section 1 deals with the introduction, gives the brief history of spread spectrum and hopping techniques. Section 2 deals with the system and channel models, the transmitter and receiver models for FHSS systems are discussed in this section. Probability of error equations are mentioned in section 3. Section 4 enlists the performance analysis of fast frequency hopping (FFH) and multicarrier frequency hopping (MCFH) techniques. Finally conclusion has drawn in section 5.

2 SYSTEM MODELLING

Consider the system is frequency hopping spread spectrum system with DBPSK modulation technique, non coherent detection and diversity order L.

2.1 Transmitter Model

Fig.1 and fig. 2 shows the transmitter block diagram for FFH and MCFH systems respectively. MCFH systems require more devices than FFH systems. However, the devices including frequency synthesizer for FFH systems are required to operate more rapidly than those for MCFH systems. The

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use of FFH system may not be feasible for high data rate systems, due to its high speed requirements. In this paper, maximal ratio combining is used to combine various diversity receptions.

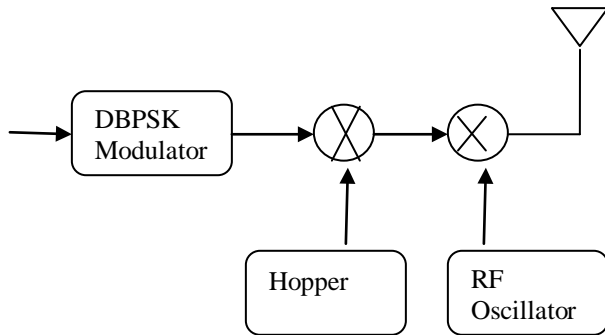


Fig.1. FFH Transmitter Block Diagram

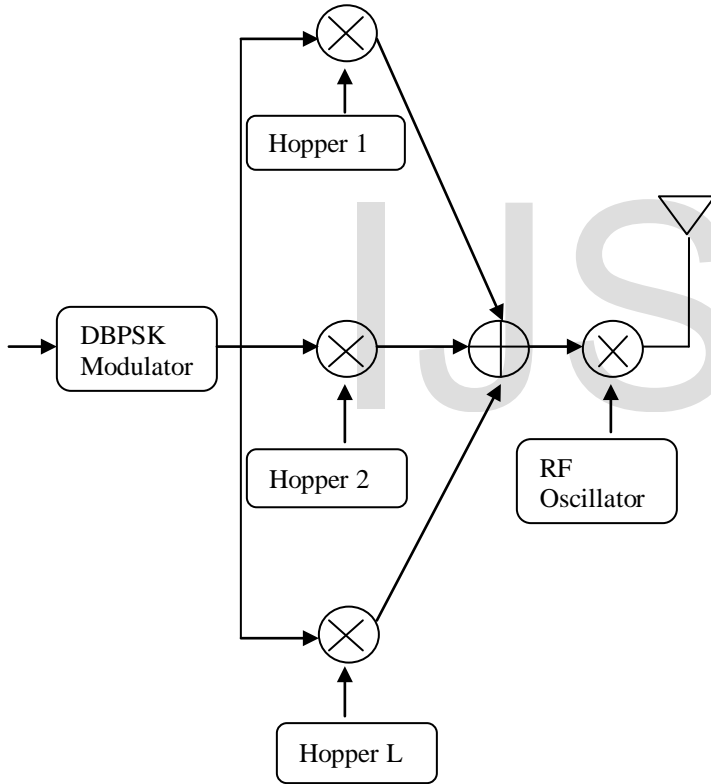


Fig.2. MCFH Transmitter Block Diagram

The complex baseband equivalent of the transmitted signal for each system can be represented as [8, 9]:

For FFH system:

$$s(t) = \sum_{m=0}^{\infty} \sum_{l=0}^{L-1} x(t) \exp[j\{2\pi f_{l,m} + \phi_{l,m} + \pi(n-1)\}] p_{T_h}(t - mT - lT_h)$$

For MCFH system:

$$s(t) = \sum_{m=0}^{\infty} \sum_{l=0}^{L-1} x(t) \exp[j\{2\pi f_{l,m} + \phi_{l,m} + \pi(n-1)\}] p_{T_h}(t - mT_h)$$

In the complex baseband equivalent representation, $x(t)$ represents the transmit baseband signal, T is the symbol duration, T_h is the hop duration. $f_{l,m}$ and $\phi_{l,m}$ are respectively, hop frequency and random phase for the l^{th} diversity transmission of the m^{th} symbol. $n \in \{1,2\}$ is the m^{th} data symbol and $p_{T_h}(t) = 1$ for $t \in (0, T_h)$ and zero otherwise.

2.2 Receiver Model

Receiver block diagrams of FFH and MCFH systems are shown in fig.3 and fig.4 respectively. The channel model is a wide-sense stationary uncorrelated scattering (WSSUS) model. The low-pass equivalent impulse response of the l^{th} diversity channel may be written as:

$$c_l(t, \tau) = \alpha_l(t, \tau) \exp[j\zeta_l(t, \tau)]$$

Where $\alpha_l(t, \tau)$ are independent and identically distributed (i.i.d) Rayleigh random processes over $[0, 2\pi]$. The autocorrelation function of the WSSUS channel is given as:

$$R_c(\Delta t, \tau, \tau') = (1/2) E [c^*(t, \tau) c(t + \Delta t, \tau')] \\ R_c(\Delta t, \tau) \delta(\tau - \tau')$$

Where $*$ denotes a complex conjugate operation. Since the channel response for each diversity transmission is assumed to be i.i.d., the autocorrelation of each channel is same. If we let $\Delta t = 0$ in $R_c(\Delta t; \tau)$, the resulting autocorrelation function $R_c(0; \tau)$ is a multipath intensity profile, and denoted as $I_c(\tau)$. Assuming that the multipath intensity profile is time invariant, $R_c(\Delta t; \tau)$ may be represented as:

$$R_c(\Delta t; \tau) = I_c(\tau) X_c(\Delta t)$$

Value of $X_c(\Delta t)$ can be taken as:

$$X_c(\Delta t) = J_0(2\pi f_d \Delta t)$$

Where J_0^* is the zeroth order Bessel function. Value of

$I_c(\tau)$ for Rayleigh fading channel is given as:

$$I_c(\tau) = \frac{(\mu / T_m) [\exp(-\mu\tau / T_m) - \exp(-\mu)]}{1 - (1 + \mu) \exp(-\mu)}$$

where μ is the decaying factor.

Assume that each of the channels corrupts the signaling waveform transmitted through it by introducing a multiplicative gain and phase shift, represented by the complex valued number g_k , and an additive noise

$n(t)$. However g_k is not constant throughout the chip duration but it can be considered constant for slow varying fading channels.

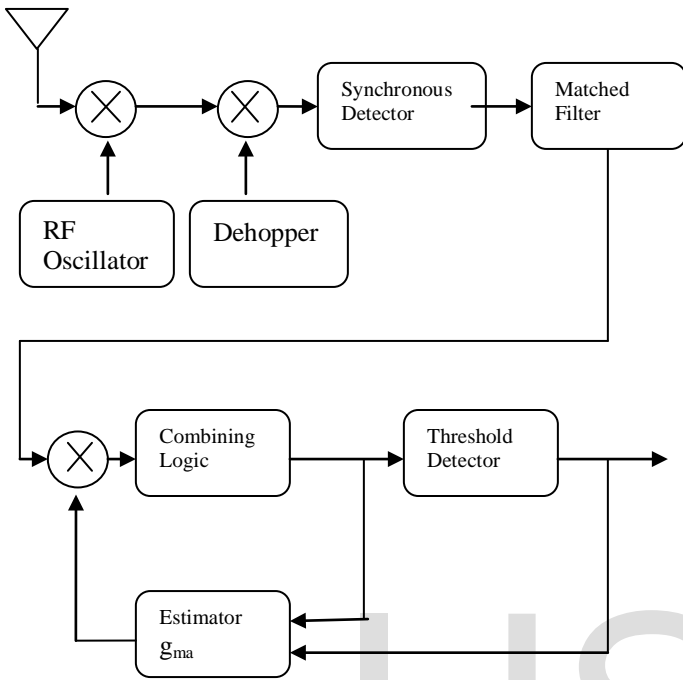


Fig.3. FFH Receiver Block Diagram

Thus, when the transmitted waveform is $s(t)$, the waveform received over the m^{th} channel is:

$$r(t) = g_m s(t) + n(t) \quad 0 \leq t \leq T_h, \quad m = 1, 2, 3, \dots, L$$

$$g_m = \frac{1}{T_h} \int_0^{T_h} \int_0^{T_h} \alpha_l(t, \tau) \exp(j\theta_l(t, \tau)) dt d\tau$$

The noises $n(t)$ are assumed to be sample functions of a stationary white Gaussian random process with zero mean and autocorrelation function $N_o \delta(\tau)$, where N_o is the value of the spectral density. These sample functions are assumed to be mutually statistically independent [10].

At the demodulator, $r(t)$ is passed through a filter whose impulse response is matched to the waveform $x(t)$. The output of the filter sampled at time $t = T_h$ is denoted as:

$$X_m = 2\xi g_m \exp[j\pi(n-1)] + N_m$$

In above equation, ξ is the transmitted signal energy per channel and N_m is the noise sample from the m^{th} filter. In order for the demodulator to decide which of the two phases were transmitted in the signalling interval $0 \leq t \leq T$, it

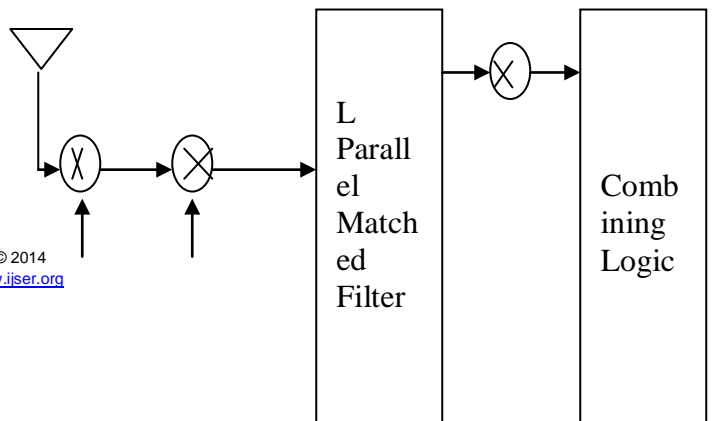
attempts to undo the phase shift introduced by each channel. In practice, this is accomplished by multiplying the matched filter output X_m by the complex conjugate of an estimate g_{ka} of the channel gain and phase shift. The result is a weighted and phase shifted sampled output from the m^{th} filter, which is then added to $L-1$ channel filters. The estimate g_{ka} of the gain and phase shift of the k^{th} channel is assumed to be derived by undoing the modulation on the information bearing signals received in previous signalling intervals. If one knew the information component contained on the matched filter output then an estimate of g_k could be obtained by properly normalizing this output. For example, the information component in the filter output is $2\xi g_k \exp[j\pi(n-1)]$, and hence, the estimate is:

$$g_{ma} = \frac{X_m}{2\xi} \exp[-j\pi(n-1)] = g_m + \frac{N_{ma}}{2\xi}$$

where $N_{ma} = N_m \exp[-j\pi(n-1)]$ and pdf of N_{ma} is identical to the pdf of N_m , an estimate that is obtained from the information bearing signal in this manner is called a clairvoyant estimate. The estimate can be improved by extending the time interval over which it is formed to include several prior signalling intervals, as a result of extending the measurement interval, the signal to noise ratio in the estimate of g_k is increased [11, 12]. The clairvoyant estimate that is obtained from the information bearing signal by undoing the modulation over the infinite past

$$\text{is: } g_{ma} = g_m + \frac{\sum_{i=1}^{\infty} c_i N_{mi}}{2\xi \sum_{i=1}^{\infty} c_i}$$

As indicated, the demodulator forms the product between g_{ma}^* and X_m , adds this to the products of the other $L-1$ channels.



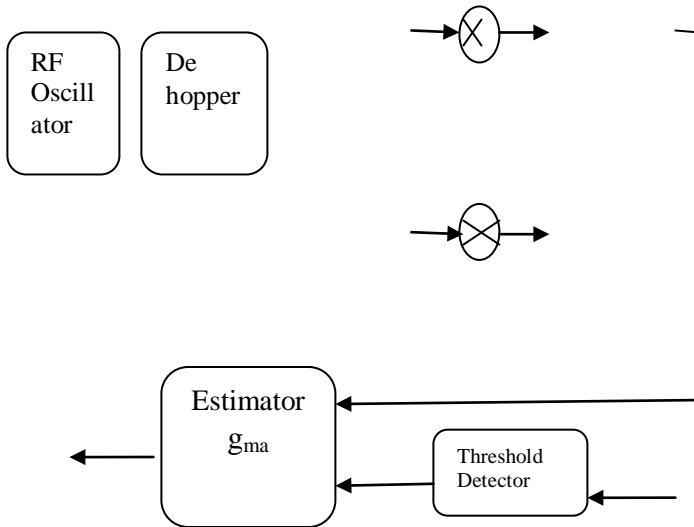


Fig.4. MCFH Receiver Block Diagram

The random variable that results is:

$$Z = \sum_{m=1}^L X_{ma} x_{ma}^* = \sum_{m=1}^L X_m Y_m^* = Z_r + jZ_i$$

Where $Y_m = g_{ma}$, $Z_r = \text{Re}(Z)$, and $Z_i = \text{Im}(Z)$ the phase of Z is the decision variable. This is simply

$$\theta = \tan^{-1} \left(\frac{Z_i}{Z_r} \right) = \tan^{-1} \left[\frac{\text{Im} \left(\sum_{m=1}^L X_m Y_m^* \right)}{\text{Re} \left(\sum_{m=1}^L X_m Y_m^* \right)} \right]$$

3 PROBABILITY OF ERROR

For probability of error equation, assumption has made that transmitted signal phase is zero i.e. $n = 1$. If desired, the probability density function (pdf) of θ conditional on any other transmitted signal phase can be obtained by translating $p(\theta)$ by the angle $\pi(n-1)$. We also assume that the complex valued number g_m , which characterize the L channels, are mutually statistically independent and identically distributed zero mean Gaussian random variables [13]. For this condition, value of $p(\theta)$ comes out to be:

$$p(\theta) = \frac{(-1)^{L-1} (1-|\mu|^2)^L}{2\pi(L-1)!} \left\{ \delta^{L-1} \left[\frac{1}{b-|\mu|^2 \cos^2(\theta-\varepsilon)} + \frac{|\mu| \cos^2(\theta-\varepsilon)}{[b-|\mu|^2 \cos^2(\theta-\varepsilon)]^{3/2}} \cos^{-1} \left(\frac{-|\mu| \cos(\theta-\varepsilon)}{b^{1/2}} \right) \right] \right\}_{b=1}$$

In this case, the probability of a binary digit error is obtained by integrating the pdf $p(\theta)$ over the range $1/2\pi < \theta < 3\pi$.

Since $p(\theta)$ is an even function and the signals are priori equally likely, this probability can be written as:

$$P_e = 2 \int_{\frac{\pi}{2}}^{\pi} p(\theta) d\theta$$

Hence, it can be easily derived that:

$$P_e = \frac{1}{2} \left[1 - \mu \sum_{m=1}^L \left(C^{2m}_m \right) \left(\frac{1-\mu^2}{4} \right)^m \right]$$

Where $\mu = \gamma / (1 + \gamma)$, and

$$\gamma = \frac{\xi}{N_o} E(|g_m|^2) \quad m = 1, 2, 3, \dots, L$$

$$E(|g_m|^2) = \frac{2}{T_h} \int_0^{T_i} \int_0^{T_i-\tau} R_c(t, \tau) \left(1 - \frac{t+\tau}{T_h} \right) dt d\tau$$

Now bit error rate (BER) is function of signal to noise ratio, diversity order, delay spread, Doppler spread, bit duration and type of diversity i.e. FFH and MCFH [14].

4 BER PERFORMANCE OF FFH AND MCFH SYSTEMS

The BER performance of FFH and MCFH systems is evaluated using probability of error equations. It can easily proved using the aforementioned equations that BER is monotonically increasing function of ratio of maximum delay spread (T_m) to hopping period. Bit duration taken is 10^{-4} and 10^{-6} second, which approximate the minimum and maximum limit of WLAN, PAN, Bluetooth systems. Also it can be easily stated that FFH is better than MCFH if time spreading is very poor and Doppler spread is very severe and further it can be emphasized that effect of delay spread is more severe than Doppler spread in both frequency hopping spread spectrum systems. As the diversity order increases, T_m / T_h ratio increases in FFH system and remains constant in MCFH system. Due to this factor improvement in performance by increasing the value of diversity order is more in MCFH system than FFH system.

5 CONCLUSION

The performance analysis of frequency hopping spread spectrum systems in slowly Rayleigh fading channels has been demonstrated and compared in this paper. It is found that fast frequency hopping techniques outperforms multicarrier frequency hopping techniques only when time spreading is very poor and Doppler spread is very severe. In all other cases multicarrier frequency hopping spread systems completely outperform fast frequency hopping spread systems up to 4db. Effect of delay spread is more severe than Doppler spread in

both frequency hopping spread spectrum systems. Improvement in performance by increasing the value of diversity order is more in multicarrier frequency hopping spread spectrum system than fast frequency hopping spread spectrum system.

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